Enhancing Mathematical Understanding Through Self-assessment and Self-Regulation of Learning: The Value of Meta-Awareness

Rosemaree Caswell
Griffith University
<rosemaree.caswell@pacific.net.au>

Steven Nisbet
Griffith University
<s.nisbet@griffith.edu.au>

This paper focuses on primary-school students' meta-awareness of their mathematical thinking and communicating of both affective and cognitive factors that enhance their learning. Meta-awareness is emphasised through engaging students in communication about their mathematical reasoning and in reflection on their levels of knowing and confidence to work mathematically. This engages students in a self-assessment and self-regulation of their own learning and valuing of the complexity of linking different mathematical concepts and different knowledge disciplines when working with mathematical contexts. The self awareness of one's learning is identified as a hidden or subliminal factor that can enhance learning and empower students to engage effectively and confidently when working mathematically.

Communication of mathematical thinking has also been identified as a key issue in mathematics learning (NCTM, 1998). Student engagement in a rich discourse of metalanguage is advocated for the effective development of mathematical conceptual understanding (Sfard, 2001). So too, studies into the effects of student engagement in metacognition have proven students' problem solving competencies are improved through awareness of mathematical reasoning (Campbell & White, 1997; Goos, Galbraith & Renshaw, 2002). Communication, however, can extend beyond students' engagement with mathematical knowledge and processes to include communication of their levels of understanding and confidence to engage with mathematical contexts. Trotman (1998) draws attention to the need for communication to extend beyond cognitive acquisition to include inter/intra-personal awareness of working mathematically.

Motivation, enjoyment and confidence to engage with mathematical concepts have been recognized as factors that impact on students' learning (McLeod, 1989). Research conducted into the affective domain focuses on attitudes, feelings and beliefs on learning (Taylor, 1992; Steele, 2001). The challenge exists to engage students in reflection that raises their consciousness of both cognitive and affective factors that affect their learning potential. In so doing, the assumption is that this conscious awareness of one's mental attitude to learning can impact positively on what Bransford, Zech, Schwartz, Barron, Vye, and The Cognition and Technology Group at Vanderbuilt (1996) term the 'Zone of Sensitivity to Instruction', in effect optimising the learning climate for each student and improving students' mathematical competencies. This research focuses on two objectives: firstly to explore and analyse students' perceptions of working mathematically, and secondly to analyse the impact of meta-awareness and self-assessment.

Theoretical Background

The basic theory of learning that underpins this research is the constructivist approach as outlined in the works of von Glasersfeld (1992) and others. Bauersfeld (1992) argues strongly for social constructivism, in which meaning is constructed through discourse and interaction. Wood, Cobb and Yackel (1992) and Sfard (1998) argue for the interactionist view that the construction of knowledge occurs not in isolation but within a social and cultural context in which discourse is a vital component in establishing an effective learning context.

The ‘Zone of Proximal Development’ as outlined by Vygotsky (1962), reflects both social constructivism and the interactionist theory of learning. In this learning theory, it is argued that children need a learning climate which recognizes both affective and cognitive factors. Bransford et al. (1996) in applying Vygotsky’s theory termed the learning climate a ‘Zone of Sensitivity to Instruction’ and argued for an approach to learning theory that recognizes the affective domain and the need for open-ended inquiry learning. The constructivist theory is usually applied to the acquisition of knowledge; however, this research expands the boundaries of constructivism to argue that children construct both affective and cognitive knowledge.

Literature Review

Research provides a clear platform for arguing that communicating mathematical understanding through metalanguage, metacognition and social discourse impacts on learning (Cobb, 1994; Sfard, 1998; Wood et al, 1992). Metalanguage is the domain specific language of mathematics. Cobb (1994) describes the language of mathematics as ‘a process of enculturation’, necessary for effective communication about working mathematically. Gawned (1990) presents a model of the development of metalanguage that recognizes the domain specific language, but also acknowledges the transition from students’ own language/real world language to accommodate the terminology of abstract mathematics.

Ellerton and Clarkson (1996) expand on Gawned’s linguistic model by creating a framework for interpreting a broad range of language factors in mathematics. Their research concludes that to engage students effectively in constructing meaning, the learning environment needs to facilitate genuine negotiation by the learners. They present the argument for classroom discourse that engages teachers and students in ‘open-ended or goal-free questions’ (Ellerton & Clarkson, 1996, pp997-1000). This argument aligns to the studies of student engagement in metacognition where open-ended questions are used to structure classroom discourse and enhance mathematical thinking. Several studies on meta-cognition have been conducted using variations of Ellerton and Clarkson’s open-ended questioning to engage students in self-reflection on their mathematical thinking (Goos et al, 2002; Campbell & White, 1997).

There is also a growing trend in research to address affective factors that impact on learning and recognize assessment practices that draw evidence from the students’ communication of their levels of confidence to engage with mathematical contexts. Trotman (1998) argues that students’ self-assessment engages students in self-awareness and self-evaluation of factors that impact on their learning. So too, Fernandez, Hadaway, and Wilson (1994) argue for the need for students to self monitor their understandings and actions. They claims

> “students’ managerial processes, including monitoring, regulating and assessing their own knowledge and actions are an important part of …problem solving abilities and must be given attention. (p. 198)

The arguments supporting student self-awareness as a factor that impacts on learning are further developed by McLeod (1989), Mandler (1989) and others. Bastick (1993) also addresses the affective domain and its impact on the learner claiming that “studies of metacognitive regulation have avoided the generation and cuing of affect and ignored other preconscious involvement" (Bastick, 1993, p. 83). Marshall (1989) claims that affect is not only a factor in learning and development but can be learned simultaneously during learning episodes or after a schema (mathematical knowledge structure) is fully formed. In effect, Marshall (1989) claims that attitudes and beliefs underpin all learning, affecting engagement in the learning context, as well as the confidence or lack thereof, which is engendered in the learner through the learning experience. Both Bastick (1993) and Marshall (1989) advocate for open cuing and communication of both affective and cognitive factors in order to raise student awareness of their learning. Thus, it can be argued that as educators, teachers should recognize both cognitive and affective factors that impact on learners and the importance of engaging the learner in communicating about their thinking.

Methodology

As the intent of the research was to document and analyse students' reflections on meta-awareness, a retroductive approach as outlined by Blaikie (2000), was adopted. The research took the form of a case study of a single primary level class. The classroom where the research study was conducted had the benefit of providing a natural setting where students were already engaged actively in communicating through metalanguage, metacognition and self-assessment. The class consisted of 27 students in a multi-age setting. The ages ranged from 9 years to 12 years. The ratio of boys and girls in the class was 15:12.

Data was collected over a ten-week period. Throughout the weekly cycle of activities, students experienced a range of mathematical tasks and varied interactions - working individually, in pairs, small groups or as whole class. Communication of understanding, problem-posing and problem solving occurred continuously during the activities. At the end of the weekly cycle of activities, students were engaged in self-reflection of their learning. This required collaborative development of 'Levels of Knowing' developed from the models of Mason and Spence (1999) and Carpenter and Lehrer (1999). The students were also engaged in regulating their own learning – choosing their level of confidence and competence to engage with a particular level of knowing. The teacher's role became a fine balance between open-ended questioning and semi-structured interviewing. Throughout the weekly activities, the teacher continuously asked specific questions about the value and use of meta-language, metacognition and meta-awareness. This effectively maintained students' awareness of factors that impacted on their learning, but allowed students to respond freely without providing a predictable answer.

Three sources of qualitative data were collected: audio-recordings/ transcripts of weekly conferences, student written reflections in journals/surveys, and models of 'Levels of Knowing' created by the class. A survey was completed at the end of the ten weeks, and provided quantitative data used to analyse the importance placed by the students on different factors in the study. The survey was compiled using a representative sample of student comments from the previous ten weeks. Students were asked to rate how much they agreed/ disagreed with each comment. This provided a general consensus of factors in the study.

Results

The effect of engaging students in meta-awareness was seen firstly in their growing confidence to respond and reflect on the learning context. Initially, there were frequent teacher prompts to encourage students to explain their thinking, but as students became familiar with the class conferences, they began to offer opinions freely and state their views on the learning. From the student responses a pattern of thinking emerged that highlighted three affective factors as primary to effective learning: enjoyment, confidence and engagement.

Enjoyment was one of the factors on which students' frequently commented. At first the students' comments simply stated whether the activity was 'fun' and they enjoyed participating. Later, they began associating fun/enjoyment with other factors such as confidence and engagement:

"It was fun, so we all got involved and enjoyed it."

"Because it was a game, everyone enjoyed it and got involved."

The links between the affective factors of enjoyment, confidence and engagement soon became associated with cognitive factors as well. Students described their competency to work mathematically in terms of their feelings and in particular their confidence to engage with the concepts.

"You need to get in and do the activity in order to learn anything. If it is fun, everyone at least tries."

"We learn better because we enjoy it."

The study also collected data on student meta-awareness through engagement in self-assessment and self-regulation. The Levels of Knowing developed by Mason and Spence (1999) focussed on knowledge, application and the ability to 'act in the moment' or effectively use knowledge in different contexts. In this study, students developed their own levels of knowing for each mathematical concept with the following key elements emerging (see Table 1):

Table 1
Key Elements in Levels of Knowing as Defined by Students

1. confidence	Individual sense of competence
2. independence	Ability to work out problems without support
3. understanding	Knowing both facts and procedures
4.application/ accuracy	Using mathematical knowledge accurately and in different contexts
5. automation	Drawing on a number of mathematical concepts and making cognitive links automatically

Students referred to confidence as the factor that was most important in determining competency. They explained confidence as the 'Yes' feeling when one knows that one knows something and can work comfortably in that zone or as Bastick (1993, p. 83) describes "the sense of knowing what one knows and how one knows it." This did not preclude students from working towards a higher level and in fact they were aware of working on a more challenging level with peer and teacher support - demonstrating

Vygotsky's 'Zone of Proximal Development'. Students then began defining their competencies on a continuum of development.

"I am confident on Level 3 but I can also do some of the next level but not really confidently yet – I still need to check with the teacher or my partner."

The next element referred to frequently was the ability to work independently. Students determined that independence meant students were aware they could think and act mathematically without the support of teacher or peers. Students' comments recognized the value of 'shared construction of knowledge', but also identified the need to work independently (see Table 2).

Table 2
Levels of Knowing – Number Concepts - Percentage

Level 5	Able to work independently to complete the activity – sorting Smarties, estimating, counting and recording as a percentage
Level 4	Confident to complete the activity but needed some advice from the teacher to record percentages accurately
Level 3	Began confidently but didn't consider all the things to do – needed advice from the teacher or ideas from other groups to calculate and record percentages
Level 2	Completed the activity but needed to ask lots of questions and check each step. Some ideas of percentage but needed help to calculate and record.
Level 1	Rushed into the activity without first observing or thinking and didn't complete it properly – may understand percentage but didn't demonstrate it.

Mathematical knowledge and understanding was always included in the 'Levels of Knowing'. There was often no differentiation between factual and procedural knowledge. Students referred mainly to the application of factual knowledge through 'doing' maths and using the metalanguage. Ellerton and Clarkson (1996) argue "assessment of mathematical understanding should involve an examination of students' work as they engage in 'real mathematics'". The students' perceptions of knowledge assumed factual knowledge was already established – knowing the mathematics involved working with the knowledge, and adding to factual knowledge with an increased understanding of the same concept.

As each 'Level of Knowing' was produced, students became more specific about the levels, showing, as Trotman (1998) identified, a growing sophistication. From the data, students indicated metalanguage was indicative of understanding but they preferred to have the freedom to use language that allowed them to express themselves most articulately and improve their mathematical understandings.

> *"It (metalanguage) is sometimes important, but you should use what you are comfortable with."*
>
> *"Maths words are very helpful and make it easy to explain things."*

Application was defined as the use of mathematical concepts in other contexts and across other fields of knowledge (i.e. the shopping activity required students to produce a shopping list to meet a specific budget and encompass the five food groups. This engaged students in working mathematically with money, percentage, decimals, measurement and calculators within the medium of advertising, and the domain of health science). Therefore, students considered a high level of knowing required confidence to use a wide range of mathematical knowledge in different situations. Raising awareness of metacognition and engaging students in discussions about the way they worked on mathematical tasks resulted in a greater understanding of 'Levels of Knowing'.

Lastly, students began referring to automation – the mental computations that occurred and frequently required the ability to use all of the above key elements. Students explained the ability to see the mathematics, make links to other mathematical knowledge in your mind and use this knowledge, whilst being aware of what you were doing, was indicative of a very competent mathematician. This is similar to the highest 'Level of Knowing' as defined by Mason and Spence (1999) as 'knowing to act in the moment'. Students' comments on meta-awareness taken from journal entries supported the research findings of Bastick (1993), and Goos et al. (2002) who claim that engaging students actively in communicating about their mathematical thinking enhances their ability to regulate their learning.

> *"I like knowing where I am and what I have to do to improve."*

> *"I like knowing what level I am on and seeing the next level. It is a challenge to reach that."*

Overall, it was obvious through the student comments that they were very aware of their learning, of factors that impacted on their learning and of the continuum of development evident in the classroom.

Conclusions

Understanding the complex nature of effective learning in mathematics most often focuses on cognitive factors and metacognition. Yet this study has revealed through the minds of the learner, the potential to improve student engagement in mathematical learning and improve student levels of understanding by engaging students effectively in meta-awareness and self-regulation.

From the study, two key points were highlighted – firstly, engaging students in meta-awareness and raising their conscious appraisal of factors that impact on their learning, resulted in affective factors being given priority. The impact of placing precedence on these factors and engaging the learner in a conscious awareness of their attitudes to working mathematically resulted in students associating attitude with engagement. From the study, it became clear that students were able to improve their own 'Zone of Sensitivity to Instruction' (Bransford et al, 1996; Vygotsky, 1962) by being aware of their attitudes and overcoming prior blockages that had built up over a period of time. By communicating their confidence to work mathematically, many students expressed a change in their attitudes to their own mathematical competence and were more willing to engage with unknown or challenging mathematical tasks (Marshall, 1989; Mandler, 1989).

The second finding from the study highlights the difference between the 'achievement/ benchmarking paradigm' and the 'developmental continuum paradigm'. As students became actively engaged in reflections about their learning and in self assessment of their mathematical competencies, it was noticed that students began to perceive their ability to work mathematically in different terms. Prior to engaging in meta-awareness and self assessment, students were familiar with an achievement paradigm in mathematics. Concepts were 'taught' and students were rated according to their competence to regurgitate the knowledge and processes they had been taught. This had lead students to perceive their ability to work mathematically within an achievement or failure system (Bauersfeld, 1992).

The effects of engaging students in self reflection of their learning involved students in observing and recording student outcomes of learning as they engaged in mathematical activities (Ellerton and Clarkson 1996). In effect, they witnessed all students engaged in working mathematically, so the sense of failure diminished and very quickly disappeared from the discourse about understanding. Instead students began to observe and verbalize that there were different levels of knowing across the class and these differences could be explained in terms of confidence, competence, levels of support required etc. So in effect, they understood the concept of the developmental continuum – all students can succeed in mathematics and achievement is defined by the 'Zone of Proximal Development' (Vygotsky, 1962) – the point along the journey where confidence to approach a mathematical task is optimal and students are engaged in an interactive, challenging learning environment. This knowledge of themselves as learners could be seen to empower students by changing their attitudes and building their confidence as risk takers.

Engaging students in meta-awareness also had the effect of changing their attitudes to their roles in the learning process. Through engaging in meta-awareness, students began regulating their own learning. By knowing the expectations of the task and the different levels of working mathematically – students began identifying the level on which they believed they were working, then actively choosing to extend themselves to a higher level. Self-regulation and responsibility for learning became the shared responsibility of teacher and student (Trotman, 1998). From an educational viewpoint the findings of this study challenge a traditional emphasis in mathematics education on 'achievement' over 'understanding'. The results of this study clearly show the pathway to achieving deeper understanding of mathematical concepts and improved communication of mathematical knowledge lies in engaging the student effectively in the learning (Bastick, 1993). Mathematics classrooms that establish meta-awareness as a priority in building Zones of Proximal Development engage and empower students as learners. The pressure to achieve is replaced with a desire to improve and the confidence to achieve on-going success as mathematicians.

References

Bastick, T. (1993). Teaching the understanding of mathematics: Using affective contexts that represent abstract mathematical concepts. In B. Atweh, C. Kanes, M.Carss & G. Booker (Eds.), *Concepts in Mathematics Education* (pp.93-99). Sydney: MERGA.

Bauersfeld, H. (1992). The structuring of the structures: Development and functioning of mathematics as a social practice. In Steffe, L. & Gale, J. (Eds.), *Constructivism in education* (pp. 137-144). Hillsdale N.J.: Erlbaum.

Blaikie, N. (2000). *Designing social research: The logic of anticipation*. Malden, MA: Blackwell.

Bransford, J., Zech, L., Schwartz, D., Barron, B., & Vye, N. (1996). Fostering mathematical thinking in middle school students: Lessons from research. In R. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 203-220). Mahwah, N.J.: Erlbaum.

Campbell, P., & White, D. (1997). Project IMPACT: Influencing and supporting teacher change in predominantly minority schools. Iin E. Fennema & B. Nelson (Eds.), *Mathematics teachers in transition* (pp. 309-355). Mahwah, N.J.: Erlbaum.

Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19-32). Mahwah, N.J.: Erlbaum.

Cobb, P. (1994). Constructivism in mathematics and science education. *Educational Researcher, 23*(7), 4.

Ellerton, N., & Clarkson, P. (1996). Language factors in mathematics teaching and learning. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook of mathematics education* (pp.987-1033). Dordrecht: Kluwer.

Fernandez, M., Hadaway, N., & Wilson, J. (1994). Problem solving – Managing it all. *The Mathematics Teacher, 87* (3), 195-199.

Gawned, S. (1990). An emerging model of the language of mathematics. In J. Bickmore-Brand (Ed.), *Language in mathematics* (pp. 27-42). Carlton, Vic.: Australian Reading Association.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Education Studies in Mathematics, 49*(2), 193-223.

McLeod, D. (1989). Beliefs, attitudes and emotions: New views of affect in mathematics education. In D.B. McLeod & V.M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 245-258). New York: Springer-Verlag.

Mandler, G. (1989). Affect and learning: Reflections and prospects. In D.B. McLeod & V.M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 237-244). New York: Springer-Verlag.

Marshall, S. (1989). Affect in schema knowledge: Source and impact. In D.B. McLeod & V.M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 49-58). New York: Springer-Verlag.

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Education Studies in Mathematics, 38*, 135-161.

National Council of Teachers of Mathematics (1998). *Principles and standards for school mathematics.* Reston, V.A.: NCTM.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Education Researcher, 27*(2), 4-13.

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Education Studies in Mathematics, 46*(3), 13-57.

Steele, D. (2001). Using sociocultural theory to teach mathematics: A Vygotskian perspective. *School Science and Mathematics, 101*(8), 404-416.

Taylor, L. (1992). Mathematical attitude development from a Vygotskian perspective', *Mathematics Education Research Journal, 4*(3), 8-23.

Trotman, S. (1998). Student self-assessment: Some insights from St Vincent. In *Transformation in Higher Education: Conference Proceedings':* Higher Education Research and Development Society of Australasia. Auckland, N.Z. {Available online] http://www2.auckland.ac.nz/cpd/HERDSA/HTML/TchLearn/Trotman.HTM [Accessed 1st August, 2004]

von Glasersfeld, E. (1992). A constructivist approach to teaching. In L. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 2-23). Hillsdale, N.J.: Erlbaum.

Vygotsky, L. (1962). *Thought and language.* Cambridge, M.A.: University Press.

Wood, T., Cobb, P., & Yackel, E. (1992). Reflections on learning and teaching mathematics in the elementary school. In L. Steffe & J. Gale (Eds.), *Constructivism in education* (pp.401-422). Hillsdale, N.J.: Erlbaum.